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CYCLIC STRESS RELAXATION OF POLYCRYSTALLINE METALS AT ELEVATED TEMPERATURE

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Abstract—The influence of a relatively small, superimposed alternating strain on the relaxation behavior of structural metals is examined from the micro-mechanics standpoint, taking into account the nature of crystallographic slip in the grain and grain interactions. The initial mean strain is allowed to be sufficiently large to include both elastic and plastic components; it thus possibly causes an instant relaxation due to plastic deformation. To facilitate the evaluation of this instantly relaxed stress state Hill's self-consistent relation for an elastoplastic polycrystal is modified from its incremental to a total form for a monotonic straining process. The self-consistent relation previously derived for static relaxation of an elastic-creep medium is also extended to the dynamic condition, and for an elastic–plastic-creep medium. Together with a set of unified constitutive equations of slip systems, these self-consistent relations are used to determine the cyclic relaxation behavior of a 0.16% carbon steel at various strain amplitudes. The results are seen to be in close agreement with the experimental data, both showing that the relaxation of mean stress could be greatly facilitated by the presence of a small strain amplitude. This accelerated process is attributed to the two factors: the additional plastic flow caused by the alternating strain and the nonlinear stress-dependency of the creep rate.

1. INTRODUCTION

The relaxation behavior of structural metals has been traditionally studied under a constant strain. Structural components, however, are commonly subjected to cyclic, or dynamic loading in service environment. Though the result of static relaxation may serve to suggest the possible trend for the dynamic response, more precise knowledge obviously has to be derived from a direct consideration of both time-dependent creep and cycle-dependent fatigue incurred in a dynamic relaxation process. Perhaps due to the additional difficulty involved in a dynamic test, very few experiments have been conducted to measure the stress relaxation under a cyclic-strain condition. But even from the limited data published thus far[1–3], there is already conclusive evidence that, when a relatively small strain amplitude is superimposed on a constant strain, the relaxation of mean stress can be greatly facilitated and, for an inelastic strain-controlled failure mode, this may result in a shorter fatigue life for the material.

Motivated by such an observation, and encouraged by a recent success in a micromechanics study of static relaxation[4], we shall examine this dynamic behavior also from the micromechanics standpoint. It was pointed out that static relaxation is a strain-free stress problem; this identification enabled us to use Eshelby's solution of ellipsoidal inclusion[5] to derive a self-consistent relation for the variation of stress and strain of a grain during a relaxation process. An important consequence of this relation is that, although the total strain of the aggregate is kept constant, the strain of a more favorably oriented grain actually continues to grow whereas that of a less favorably oriented grain continues to decrease. The stress of the former was also shown to relax faster than that of the aggregate, and conversely for the latter its stress relaxes slower. These micromechanical principles will be extended from the static to dynamic condition in this paper.

To improve the applicability of the developed theory, we shall also lift the restriction inherently imposed in [4] that the initial response be purely elastic, and allow the applied mean strain to be sufficiently large to include both elastic and plastic components. Stress relaxation under this condition then could take place in two distinctive ways: the instant relaxation due to plastic deformation and the time-dependent relaxation due to cyclic straining. Following the spirit of self-consistent approach, the instant relaxation may be evaluated by Hill's relation[6], which will be modified to a simpler form suitable for a monotonic straining process. Hill's model could account for the decreasing constraint power of a yielded matrix, and is known to be more rigorous than the original model of Kröner[7] and Budiansky and Wu[8]. Since the polycrystal now can deform plastically, the extension of self-consistent relation to the dynamic condition will also be accomplished in the context of an elastic–plastic-creep medium, not just of an elastic-creep medium discussed in [4].

We shall limit our consideration to the temperature range below about half the melting point; crystallographic slip then may be taken to be the predominant mechanism of inelastic deformation for load-carrying structural members. Within the framework of small deformation the elastic anisotropy of the grain and grain rotation will be neglected for simplicity. The developed theory finally will be applied, in conjunction with a set of unified constitutive equations of slip systems, to predict both static and dynamic relaxation behavior of a carbon steel, and to examine how a small strain amplitude may influence the relaxation of the mean stress. To place this micromechanical theory in proper perspective, theoretical results will also be compared with the experimental data.

2. MODIFICATION OF HILL'S SELF-CONSISTENT RELATION FOR A MONOTONIC STRAINING PROCESS

Consider an ellipsoidal inclusion (grain) embedded in an infinitely extended matrix (aggregate). The stress and total strain of the grain are denoted by σ and ϵ, respectively, and those of the aggregate by the corresponding barred (averaging) quantities $\bar{\sigma}$ and $\bar{\epsilon}$. Under an incremental deformation Hill's self-consistent relation can be written as

$$d\epsilon - d\bar{\epsilon} = -\mathbf{M}^*(d\sigma - d\bar{\sigma}), \tag{2.1}$$

where $\mathbf{M}^*$ is the constraint compliances tensor of the matrix, related to Eshelby's transformation matrix $\mathbf{S}$ through

$$(\mathbf{I} - \mathbf{S})\mathbf{M}^* = \mathbf{S}\mathbf{M}, \tag{2.2}$$

$\mathbf{M}$ and $\mathbf{I}$ being the tangent compliances and the fourth-rank identity tensor, respectively.

Since $\mathbf{M}$ is anisotropic in plastic deformation, the corresponding $\mathbf{S}$ is not always known, nor are the components of $\mathbf{M}^*$. The application of (2.1) and (2.2) is, in fact, quite complicated. Even for the simplest case of pure tension, under which $\mathbf{M}$ possesses a transverse isotropy, Hutchinson's study[9] already indicates that the numerical calculation involved in Hill's model is very elaborate.

To overcome such a difficulty, Berveiller and Zaoui[10] first attempted to modify Hill's incremental relation to a total form for a monotonic, stress-controlled process; then, for an initially isotropic polycrystalline aggregate, the "secant" moduli would still be isotropic in the plastic state. This modification employs the concept of Hencky–Nadai's deformation theory for the overall elastic–plastic response, but the secant moduli are calculated by the self-consistent method in polycrystalline plasticity. With the plastic constraint factor constantly adjusted for increasing stress and strain state, their modification was recently applied to calculate the stress–strain curve of a polycrystalline aggregate, and was shown to provide a much closer approximation to Hill's original model than Kröner–Budiansky–Wu's[11]. The difficulty is thus removed, while the essence of Hill's scheme, which characterizes the decreasing constraint power of an already-yielded matrix, is nevertheless preserved.

On the same footing, we may modify (2.1) and (2.2) to a total form for a monotonic, strain-controlled process, and write

$$\epsilon - \bar{\epsilon} = -\mathbf{M}_s^*(\sigma - \bar{\sigma}), \tag{2.3}$$

in parallel to (2.1), where the subscript s refers to the "secant" property. Now since the total response with respect to the undeformed state is assumed to be isotropic, the total stress and strain of the aggregate can be related by the secant shear modulus μ_s and secant Poisson's ratio v_s. Then, by analogy of Hill's $\mathbf{M}^*$ in polycrystalline elasticity, $\mathbf{M}_s^*$ reduces to merely two components; hydrostatic and deviatoric, respectively,

$$\mathbf{M}_s^* = \frac{1}{2\mu_s}\left(\frac{1}{2}, \frac{\beta_s}{1-\beta_s}\right), \tag{2.4}$$

for a spherical grain, where $\beta_s = 2(4 - 5v_s)/15(1 - v_s)$. Due to the assumed elastic isotropy for the grain and plastic incompressibility, only the deviatoric component of $\mathbf{M}_s^*$ remains effective in (2.3). If we further express μ_s in terms of the elastic shear modulus μ by

$$\mu_s = q\mu, \tag{2.5}$$

the self-consistent relation (2.3) may be simplified to

$$\epsilon = \bar{\epsilon} - (\sigma - \bar{\sigma})/B\mu, \tag{2.6}$$

where

$$B = q(7 - 5v_s)/(4 - 5v_s). \tag{2.7}$$

Alternatively, in terms of plastic strains ϵ^p and $\bar{\epsilon}^p$, eqns (2.6) and (2.7) can be written as

$$\epsilon = \bar{\epsilon} + C(\epsilon^p - \bar{\epsilon}^p), \tag{2.8}$$

$$C = 2(4 - 5v_s)/[q(7 - 5v_s) + 2(4 - 5v_s)]. \tag{2.9}$$

Because of plastic incompressibility, μ_s and v_s are not independent; indeed,

$$q = (0.5 - v_s)(1 + v)/(0.5 - v)(1 + v_s), \tag{2.10}$$

where v is the Poisson ratio.

Thus under monotonic straining, Hill's self-confident relation (2.1) is modified to a simpler form (2.6) or (2.8), in which the iterative procedure would involve the assumption of only one parameter v_s, lying between $v \leq v_s < 0.5$. This modified relation may be used to calculate the instantly relaxed stress state and the corresponding plastic strain at a given $\bar{\epsilon}$.

3. SELF-CONSISTENT RELATION FOR THE DYNAMIC RELAXATION OF AN ELASTIC-PLASTIC-CREEP POLYCRYSTAL

Following the instant relaxation due to plastic deformation, the time-dependent process due to static or cyclic straining may now take place. To establish the corresponding micromechanics principle we first recall the self-consistent relations for static relaxation in an elastic-creep medium[4]. Over a time-increment dt, if the creep strain increments of a spherical grain (inclusion) and of the aggregate (matrix) are denoted respectively by $d\epsilon^c$ and $d\bar{\epsilon}^c$, stress relaxation for the grain and the aggregate under a fixed total strain $\bar{\epsilon}$ has been shown to be

$$d\sigma = -2\mu(1 - \beta)\, d\epsilon^c - 2\mu\beta\, d\bar{\epsilon}^c, \tag{3.1}$$

$$d\bar{\sigma} = -2\mu\, d\bar{\epsilon}^c, \tag{3.2}$$

where $\beta = 2(4 - 5v)/15(1 - v)$. We now examine whether these relations would remain valid in an elastic–plastic-creep medium, and also seek to extend it to the dynamic state.

Under static relaxation, as shown in the schematic diagram, Fig. 1, strain amplitude $\bar{\epsilon}_a = 0$ and stress relaxes vertically downward from point A on the elastic–plastic stress–strain curve. Moreover, the generated creep strain will cause further work hard-

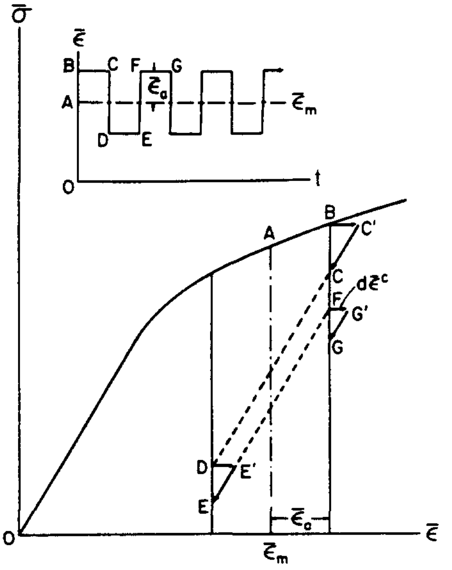

Fig. 1. Schematic diagram of dynamic stress relaxation under cyclic straining.

ening, giving rise to a higher flow stress than point *A* at any subsequent time. The aggregate and naturally all its constituent grains will always be in elastic-creep state during the static relaxation process, despite that they are potentially capable of deforming plastically. The self-consistent relations (3.1) and (3.2) thus will still remain valid in an elastic–plastic-creep medium.

Under dynamic relaxation, each strain increment is accompanied instantaneously by a stress increment. Though in an elastic-creep medium such an increment is readily determinable from the elastic stress–strain relation, possible plastic flow in each strain increment will have to be checked for an elastic–plastic-creep medium. Fortunately under cyclic straining with a constant strain amplitude, additional plastic deformation, other than the one induced by the mean strain, can occur only during the first cycle. This can be easily visualized again from Fig. 1 which indicates that, for a square wave pattern, *AB* is the only straining path that may cause the additional plastic flow. The horizontal strain segments *BC'*, *DE'* and *FG'* represent the creep strain increments in each time interval and, following the incremental creep, stress is instantly relaxed to maintain the controlled total strain. Subsequent variations of the total strain are seen to be purely elastic, since they all lie below the elastic–plastic stress–strain curve. The additional stress relaxation associated with *AB* of course could be calculated by the modified Hill's relation (2.6) or (2.8) by setting $\bar{\epsilon} = \bar{\epsilon}_B$, minus the value obtained early with $\bar{\epsilon} = \bar{\epsilon}_m$, the mean strain.

If the cyclic strain involves a decreasing strain amplitude, again only during the first cycle may additional plastic deformation be caused; on the contrary if the amplitude continues to rise, plastic deformation may occur in subsequent cycles if the increase is sufficiently rapid.

Thus for the case of constant strain amplitude, which will be specifically considered later, each strain increment in subsequent cycles is simultaneously accompanied by a stress increment governed by the elastic relation. In terms of the Lamé constants μ and λ, the self-consistent relations (3.1) and (3.2) then can be extended to the dynamic condition:

$$d\sigma_{ij} = -2\mu(1-\beta)\,d\epsilon^c_{ij} - 2\mu(\beta\,d\bar{\epsilon}^c_{ij} - d\bar{\epsilon}_{ij}) + \lambda\delta_{ij}\,d\bar{\epsilon}_{kk}, \tag{3.3}$$

$$d\bar{\sigma}_{ij} = -\,2\mu(d\bar{\epsilon}^c_{ij} - d\bar{\epsilon}_{ij}) + \lambda\delta_{ij}\,d\bar{\epsilon}_{kk}, \tag{3.4}$$

where $d\bar{\epsilon}_{ij}$ is the strain increment applied over dt and δ_{ij} Kronecker's delta. It should be noted that $d\epsilon^c_{ij}$ and $d\bar{\epsilon}^c_{ij}$ are not caused by $d\sigma_{ij}$ or $d\bar{\sigma}_{ij}$, but rather by σ_{ij} and $\bar{\sigma}_{ij}$, respectively, over the time-increment. When a strain increment is abruptly applied, such as *CD* or *EF* in Fig. 1, the corresponding creep strain increment will be zero because there is no time for it to develop. Of course if the external strain is kept constant over dt, such as BC or DE in Fig. 1, $d\bar{\epsilon}_{ij}$ will be zero and these two relations reduced to (3.1) and (3.2). Equations (3.3) and (3.4) enable one to determine the dynamic relaxation behavior of a polycrystalline aggregate in a self-consistent manner.

4. CONSTITUTIVE EQUATIONS OF SLIP SYSTEMS

To calculate the instant relaxation due to plastic deformation by means of the modified relation (2.6) or (2.8), we need to supplement it with a constitutive equation for the slip system. Under single slip, the slip behavior in most metallic crystals could be represented by the modified Ludwik equation

$$\tau = \tau_0 + h \cdot (\gamma^p)^n, \tag{4.1}$$

where τ is the current flow stress at the plastic slip strain γ^p, τ_0 the initial flow stress, and h and n two other single crystal constants. The second term on the right represents the active hardening of a slip system.

Hardening not only exists in active slip systems, it also occurs to latent ones and causes latent hardening. The amount of latent hardening is related to the active one through the latent hardening coefficient. To evaluate this coefficient the nature of major hardening mechanisms was analyzed by Weng[12]. It was found that some mechanisms, such as dislocation tangles and cell structures, are statistically symmetric and cause an isotropic hardening, while other mechanisms, such as dislocation pile-ups, are directional and their long-range stress field would lead to the kinematic hardening. A mixed hardening law which embodies both isotropic and kinematic components and which is characterized by "the degree of isotropy in work hardening α", has been introduced.

Based on this hardening law and incorporating the nonlinear active hardening (4.1), the flow stress of the i th slip system under multiple slip can be written as

$$\overset{(i)}{\tau} = \tau_0 + \sum_j [\alpha + (1-\alpha)\cos\overset{(i,j)}{\theta}\cos\overset{(i,j)}{\phi}] \cdot h \cdot (\overset{(j)}{\gamma^p})^n, \tag{4.2}$$

where $\overset{(i,j)}{\theta}$ represents the angle between the slip directions of the i th and j th slip systems, and $\overset{(i,j)}{\phi}$ between their slip plane normals. It is evident that when $\alpha = 1$, the mixed hardening law reduces to Taylor's isotropic hardening[13] and that when $\alpha = 0$, it coincides with Prager's kinematic hardening (see [14]). Under single slip, regardless of the value of α, eqn (4.1) is restored from (4.2) for the active slip system. The term inside the bracket is the latent hardening coefficient.

Following plastic deformation, the calculation of subsequent time-dependent relaxation also requires a constitutive equation to specify the creep activity of slip systems so that $d\epsilon^c_{ij}$ and $d\bar{\epsilon}^c_{ij}$ in (3.3) and (3.4) may be evaluated at each instant. The creep rate of each slip system consists of two parts: the steady $\dot{\gamma}^c_s$ and the transient $\dot{\gamma}^c_t$. For most metallic crystals the steady creep rate is a nonlinear function of its resolved shear stress, and so is the transient rate which decreases as a result of active and latent hardening. Since prior plastic deformation in active slip systems also causes work hardening, it will affect the subsequent creep; a unified description of the transient creep rate therefore should be adopted. Following the constitutive equation suggested in [11], which was also based on the mixed hardening law cited above, we write the steady and transient creep rates of the i th slip system as

$$\overset{(i)}{\dot{\gamma}}{}^c_s = \kappa \overset{(i)}{\tau}{}^{\lambda}_s, \tag{4.3}$$

$$\overset{(i)}{\dot{\gamma}}{}^c_t = \eta \left\{ \zeta \overset{(i)}{\tau}{}^{\lambda}_s - \sum_j [\alpha + (1-\alpha)\cos\overset{(i,j)}{\theta}\cos\overset{(i,j)}{\phi}](\overset{(j)}{\gamma^p} + \overset{(j)}{\gamma^c}) \right\}, \tag{4.4}$$

where τ_s is the Schmid, or resolved shear stress of a slip system, γ^c the current total creep strain, and κ, λ, η and ζ four additional single crystal constants (please differentiate this λ from the Lamé constant). Among these four constants λ characterizes the stress-dependence, κ and ζ, together with λ, represent the magnitudes of steady creep rate and of transient creep, respectively, and η the decreasing rate of $\dot{\gamma}^c_t$ with increasing creep strain. The effect of prior plastic deformation on subsequent creep is considered by introducing γ^p to the newly generated γ^c to stand for the hardening.

5. SELF-CONSISTENT CALCULATIONS

With these self-consistent relations to account for the grain interaction and constitutive equations to specify the inelastic activities in each grain, both instant and time-dependent relaxation of the aggregate may now be calculated. We first consider the instant part due to plastic deformation and then the time-dependent relaxation due to cyclic straining.

To use the modified Hill's relation to calculate the relaxed stress state $\bar{\sigma}_{ij}$ at a given $\bar{\epsilon}_{ij}$, an iterative process is necessary. First we assume a value for ν_s. The stress then follows from the isotropic relation

$$\bar{\sigma}_{ij} = 2\mu_s\bar{\epsilon}_{ij} + \lambda_s\delta_{ij}\bar{\epsilon}_{kk}, \tag{5.1}$$

where, with the assumed ν_s, μ_s is given by (2.5) and (2.10), and the other "secant" Lamé constant λ_s follows from

$$\lambda_s = 2\nu_s\mu_s/(1 - 2\nu_s). \tag{5.2}$$

The total strain of the grain ϵ_{ij} consists of two parts. Its elastic component is directly related to σ_{ij} and its plastic component is contributed by the slip strain of active slip systems,

$$\epsilon^p_{ij} = \sum_k \overset{(k)}{\nu_{ij}}\overset{(k)}{\gamma^p}, \tag{5.3}$$

where ν_{ij} is the Schmid factor tensor of a slip system, given by

$$\nu_{ij} = \frac{1}{2}(b_in_j + b_jn_i), \tag{5.4}$$

b_i and n_j being the unit slip direction and slip plane normal, respectively. After rearranging (2.6), or from (2.8), we arrive at

$$\sigma_{ij} = \frac{2}{B+2}\left[\bar{\sigma}_{ij} + \mu B\left(\bar{\epsilon}_{ij} + \frac{\nu}{1-2\nu}\delta_{ij}\bar{\epsilon}_{kk} - \sum_k \overset{(k)}{\nu_{ij}}\overset{(k)}{\gamma^p}\right)\right]. \tag{5.5}$$

The Schmid or resolved shear stress of a slip system is related to σ_{ij} through

$$\tau_s = \nu_{ij}\sigma_{ij}. \tag{5.6}$$

For an active slip system its τ_s is equal to its current flow stress given by (4.2),

$$\tau_s = \tau, \tag{5.7}$$

and for an inert one, $\tau_s < \tau$. Assuming one set of potentially active systems to be truly active in a crystal and, with (4.2), (5.5) and (5.6), we obtain a system of equations from (5.7) for the unknown plastic strain, γ^p, of the active slip systems. After γ^p is solved, the yield condition of inactive systems, $\tau_s < \tau$, should be checked. If this inequality holds for all of them, the assumed set is a correct one; otherwise a new set should be selected. This process is repeated until the correct set is found for each grain orientation and, with the calculated γ^p the corresponding σ_{ij} is determined from (5.5).

The average of σ_{ij} over all orientations gives rise to a new $\bar{\sigma}_{ij}$. Symbolically we write

$$\bar{\sigma}_{ij} = \{\sigma_{ij}\}, \tag{5.8}$$

where, and hereafter, the curly brackets represent such an average. If this newly calculated $\bar{\sigma}_{ij}$ is equal to the original $\bar{\sigma}_{ij}$ derived from (5.1), the assumed ν_s is the correct one and the solution to instant relaxation due to plastic deformation is obtained. If not, with this newly calculated $\bar{\sigma}_{ij}$ a new ν_s is estimated at the given $\bar{\epsilon}_{ij}$. This process is to be repeated until the calculated $\bar{\sigma}_{ij}$ is equal, or sufficiently close to the original $\bar{\sigma}_{ij}$.

The corresponding values of σ_{ij}, τ_s and γ^p in each grain will serve as the initial condition for the subsequent calculation of time-dependent relaxation.

Though an iterative process was required for the determination of instant relaxation, it is not for the time-dependent behavior. From (3.4), incremental stress relaxation depends on $d\bar{\epsilon}^c_{ij}$, which is given by the average of $d\epsilon^c_{ij}$ over all grain orientations. Thus,

$$d\bar{\epsilon}^c_{ij} = \{d\epsilon^c_{ij}\}. \tag{5.9}$$

The incremental creep strain in turn is contributed by the incremental slip strain of slip systems over a time interval dt,

$$d\epsilon^c_{ij} = \sum_k \overset{(k)}{\nu_{ij}} \, d\overset{(k)}{\gamma^c}, \tag{5.10}$$

where

$$d\gamma^c = (\dot{\gamma}^c_s + \dot{\gamma}^c_t)\, dt, \tag{5.11}$$

which, at a given τ_s and $\gamma^p + \gamma^c$, can be found from the constitutive equations (4.3) and (4.4).

Together with any incremental straining $d\bar{\epsilon}_{ij}$, the calculated $d\epsilon^c_{ij}$ and $d\bar{\epsilon}^c_{ij}$ are used to determine the incremental stress relaxation $d\sigma_{ij}$ and $d\bar{\sigma}_{ij}$ by the self-consistent relations (3.3) and (3.4). These increments then provide a new stress state for the next time-increment. With the new τ_s and $\gamma^p + \gamma^c$ the steady and transient creep rates of a slip system may be calculated again, leading to another stress relaxation. Following the pattern of external straining this process may be repeated until the entire relaxation history is obtained.

6. PREDICTION OF THE DYNAMIC RELAXATION BEHAVIOR OF A CARBON STEEL AND COMPARISON WITH EXPERIMENTS

In order to assess the validity of the developed theory we finally applied it to calculate the static and dynamic relaxation behavior of a 0.16% carbon steel at 450°C, and compared the results with Suzuki's experimental data[3].

Steel at the tested temperature has a body-centered-cubic lattice structure. Since Suzuki's experiments were conducted under pure tension alone, the tensile axis in our polycrystal model was chosen to lie within the standard triangle of stereographic projection. As shown in Fig. 2, twenty-five orientations were chosen, with a successive incremental rotation of 7.5°. There are three exceptions: $\psi = \omega = 15°$, $\psi = \omega = 22.5°$, and $\psi = 45°$, $\omega = 37.5°$, which originally lie outside the triangle but are brought back to lie on the line of two-fold symmetry with a reduced ω at the same ψ. As far as tensile loading is concerned, this polycrystal model is more refined than the ones previously suggested[4, 11], which, being intended for combined-loading calculations, involved successive rotations of 18° along three axes.

Six $\{110\}$ slip planes exist in a b.c.c. crystal, each having two $\langle 111 \rangle$ slip directions. Although slip on $\{112\}$ and $\{123\}$ planes have been occasionally observed, their occurrence requires a higher critical shear stress and is with less certainty. For simplicity our numerical

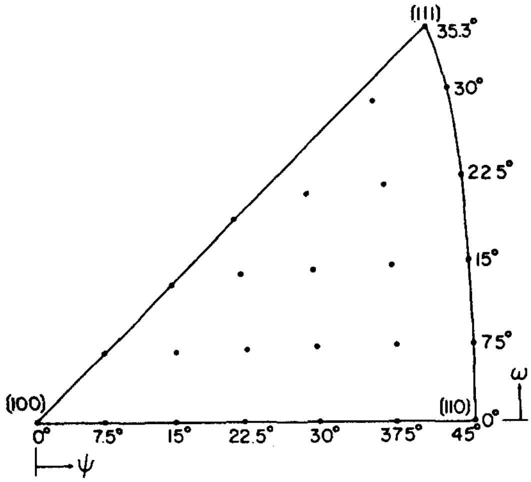

Fig. 2. Stereographic projections of the tensile axis in the standard triangle.

calculations only considered the twelve $\{110\}$ $\langle 111 \rangle$ slip systems. At the test temperature Young's modulus of the steel is $E = 176.8$ GPa and with $\nu = 1/3$, its shear modulus is $\mu = 66.3$ GPa.

Following the concept of inverse method as suggested by Lin *et al.* [15] in plasticity and Weng[16] in creep, the single crystal constants were determined by the simulation of static relaxation of the polycrystal. Three static relaxations were reported by Suzuki: $\bar{\epsilon} = 0.15, 0.25$ and 0.40%, each involving both instant relaxation due to plastic deformation and time-dependent relaxation due to creep. The instantly relaxed stress states corresponding to these three strains were found from his Fig. 4 to be: $\bar{\sigma} = 79.5$, 93.2 and 107.9 MPa, in turn. These three data points allowed us to use the modified Hill's relation to derive the three single crystal constants in plasticity: $\tau_0 = 14.72$ MPa, $h = 30.79$ MPa and $n = 0.28$, assuming an isotropic hardening ($\alpha = 1$) for the crystal. The tensile elastic–plastic stress–strain curve of the polycrystal as generated from these three data points was plotted in Fig. 3. The system of nonlinear simultaneous equations for the plastic slip strains γ^p was solved by the Newton–Bailey method, in conjunction with the quadratic Taylor polynomials and Gauss–Jordan direct elimination. In Suzuki's relaxation experiments only the tensile stress component was applied to control the constant or cyclic tensile strain; this boundary condition was imposed in our numerical calculations so that the theoretical results could be meaningfully compared with his test data.

As pointed out in [4, 11], the precise value of α may be determined from an additional experimental result involving reversed or non-radial inelastic deformation, but for monotonic deformation either isotropic or kinematic hardening may be equally well used. Suzuki's data are not sufficient to determine the value of α and, within the scope of our present objective, neither is it necessary to have such a precise value. We therefore chose the simplest case of isotropic hardening ($\alpha = 1$) in our computations, not only for plasticity but also for creep.

The other four single crystal constants in creep were determined by the simulation of two static relaxation data: $\bar{\epsilon} = 0.15$ and 0.40%, by means of the self-consistent relations (3.1) and (3.2) and the constitutive equations (4.3) and (4.4). The results are $\kappa = 4.71 \times 10^{-12}$, $\lambda = 3.90$, $\eta = 0.045$ and $\zeta = 8.61 \times 10^{-9}$, where stress and time are in the units of MPa and hour, respectively. In these simulations one should note the unique property associated with each constant. The time increment dt was taken to be 0.004 hr, in line with the duration of strain amplitude for the following dynamic cases. As shown in Figs. 4 and 5, the data points of static relaxation are well simulated by this micromechanical theory.

Using these derived single crystal constants we applied the modified Hill's equation (2.6) and the extended self-consistent relations (3.3) and (3.4) to predict the dynamic

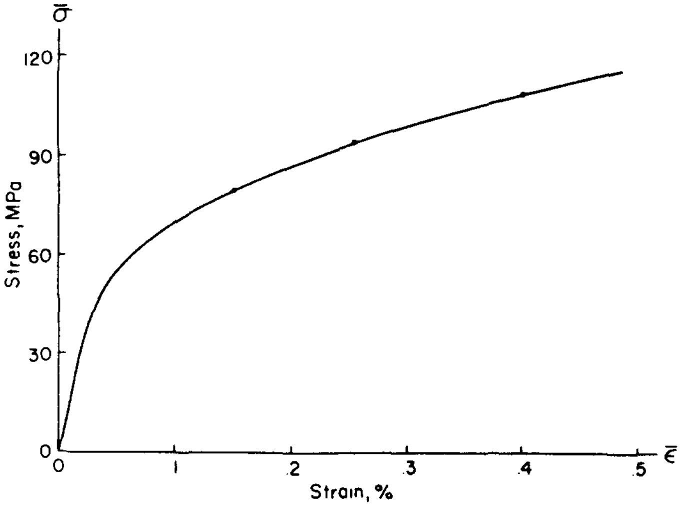

Fig. 3. Derived elastic–plastic stress–strain relation of a 0.16% carbon steel at 450°C.

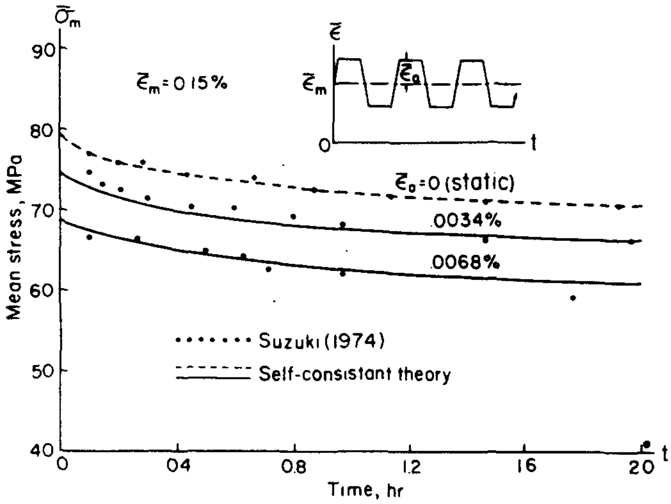

Fig. 4. Static and dynamic relaxation of mean stress of a 0.16% carbon steel at 450°C. The mean strain is 0.15%.

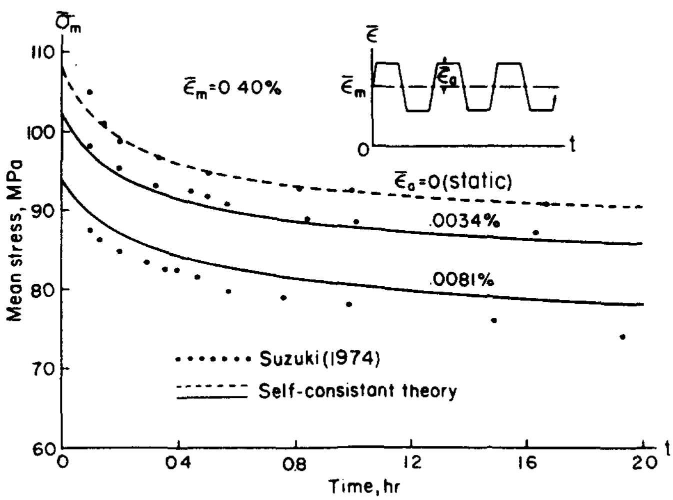

Fig. 5. Static and dynamic relaxation of mean stress of a 0.16% carbon steel at 450°C. The mean strain is 0.40%.

relaxation behavior of the same steel at several strain amplitudes. The cyclic straining process, as shown in the insert of Figs. 4 and 5, follows a symmetric trapezoidal pattern, with $\Delta t_1 = 0.00333$ hr (12 sec) and $\Delta t_2 = 0.00097$ hr (3.5 sec). In the numerical calculation the instant stress relaxation was reckoned by setting $\bar{\epsilon} = \bar{\epsilon}_m + \bar{\epsilon}_a$, since the creep strain produced during the first $\Delta t_2/2$ is negligible. For the subsequent time-dependent relaxation the brief linear transitional part (during Δt_2) was replaced by a constant strain equal to the mean strain and, following the straining history, the time increment dt was taken to be Δt_1 or Δt_2 accordingly (except the first half-cycle, in which dt was set to be 0.00111 hr to assure convergence). The results of dynamic relaxation with alternating strains 0.0034 and 0.0068% at the mean strain 0.15% are shown in Fig. 4, and those with alternating strains 0.0034 and 0.0081% at the mean strain 0.40% are shown in Fig. 5. Also depicted in these figures are corresponding experimental data. It is seen that the dynamic relaxation behavior of the carbon steel are reasonably well predicted by the theory. In addition, as mentioned in the opening section, the relaxation of mean stress indeed can be greatly facilitated by imposing a relatively small strain amplitude. Such an acceleration process is attributable to the two factors: the additional stress relaxation due to plastic deformation caused by the alternating strain in the first cycle, and the nonlinear stress-dependency of both transient and steady creep.

7. CONCLUSIONS

A micromechanical theory, which considers the deformation mechanism in each grain and grain interactions, has been proposed to study the dynamic relaxation behavior of structural metals at elevated temperature. The initial plastic deformation, transient and steady creep, and the heterogeneous stress and strain fields induced by different grain orientations, have all been considered in the derivation of instant and time-dependent stress relaxation.

In order to calculate the instantly relaxed stress state due to plastic deformation, Hill's self-consistent relation was modified from its incremental to a total form under monotonic straining, so that the iterative process in numerical calculations would involve the assumption of only one parameter. To derive the time-dependent relaxation, the self-consistent relations originally derived for static relaxation of an elastic-creep medium was extended to the dynamic case, and for an elastic–plastic-creep medium. Under cyclic straining with a constant amplitude, it was shown that additional plastic deformation, other than the one caused by the mean strain, could take place only during the first cycle, and that the constraint of the matrix on the inclusion in the auxiliary problem during subsequent cyclic straining is purely elastic, not elastic–plastic. The modified Hill's and these extended self-consistent relations are used in conjunction with a set of unified constitutive equations of slip systems.

The established theory was applied to study the dynamic relaxation behavior of a 0.16% carbon steel at 450°C. Following the concept of inverse method, single crystal constants were first determined from the data points of static relaxation, and the derived constants were then used to predict the dynamic relaxation of the same polycrystal at various strain amplitudes. The theoretical results are in good agreement with the experimental data, both indicating that the relaxation of mean stress can be greatly enhanced by a relatively small, superimposed strain amplitude.

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